AVIRAL
JEE | NEET | FOUNDATIONS

WEEKLY TEST SOLUTION TYJ – 02 - TEST – 05 18 AUGUST 2019
PHYSICS & MATHEMATICS

1. (b) Power $=\frac{\text{Work}}{\text{Time}}=\frac{ML^2T^{-2}}{T}=ML^2T^{-3}$

2. (b) Angular momentum = $mvr=MLT^{-1}\times L=ML^2T^{-1}$

3. (b) $F=\frac{Gm_1m_2}{d^2}\Rightarrow G=\frac{Fd^2}{m_1m_2}$

$\therefore\ [G]=\frac{[MLT^{-2}][L^2]}{[M^2]}=[M^{-1}L^3T^{-2}]$

4. (b) Angular momentum = mvr

$=[MLT^{-1}][L]=[ML^2T^{-1}]$

5. (c) $E=h\nu\Rightarrow[ML^2T^{-2}]=[h][T^{-1}]\Rightarrow[h]=[ML^2T^{-1}]$

6. (a) Momentum = $mv=[MLT^{-1}]$

Impulse = Force $\times$ Time = $[MLT^{-2}]\times[T]=[MLT^{-1}]$

7. (b) Pressure = $\frac{\text{Force}}{\text{Area}}=\frac{\text{Energy}}{\text{Volume}}$ = $ML^{-1}T^{-2}$

8. (d) $[h]=[\text{Angular momentum}]=[ML^2T^{-1}]$

9. (a) By principle of dimensional homogenity $\left[\frac{a}{V^2}\right]=[P]$

$\therefore\ [a]=[P][V^2]=[ML^{-1}T^{-2}]\times[L^6]$ = $[ML^5T^{-2}]$

10. (c) Let $v^x=kg^y\lambda^z\rho^\delta$. Now by substituting the dimensions of each quantities and equating the powers of M, L and T we get $\delta=0$ and $x=2, y=1, z=1$.

11. (b) From the principle of homogenity $\left(\frac{x}{v}\right)$ has dimensions of T.

12. (a) $Q=[ML^2T^{-2}]$ (All energies have same dimension)

13. (a) By substituting the dimension of each quantity we get $T=[ML^{-1}T^{-2}]^a[L^{-3}M]^b[MT^{-2}]^c$

By solving we get a = – 3/2, b = 1/2 and c = 1

14. (b) $v\propto g^ph^q$ (given)

By substituting the dimension of each quantity and comparing the powers in both sides we get $[LT^{-1}]=[LT^{-2}]^p[L]^q$

$\Rightarrow\ p+q=1,\ -2p=-1, \therefore\ p=\frac{1}{2}, q=\frac{1}{2}$

15. (a) Power = $\frac{\text{Energy}}{\text{Time}}$

31. (b) We have, $\tan\alpha=\frac{m}{m+1}$ and $\tan\beta=\frac{1}{2m+1}$

We know $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$$=\frac{\frac{m}{m+1}+\frac{1}{2m+1}}{1-\frac{m}{(m+1)}\frac{1}{(2m+1)}}=\frac{2m^2+m+m+1}{2m^2+m+2m+1-m}$$

$$=\frac{2m^2+2m+1}{2m^2+2m+1}=1\Rightarrow\tan(\alpha+\beta)=\tan\frac{\pi}{4}$$

Hence, $\alpha + \beta = \frac{\pi}{4}$.

Trick : As $\alpha + \beta$ is independent of m, therefore put $m = 1$, then $\tan\alpha = \frac{1}{2}$ and $\tan\beta = \frac{1}{3}$. Therefore,

$\tan(\alpha + \beta) = \frac{(1/2)+(1/3)}{1-(1/6)} = 1$. Hence $\alpha + \beta = \frac{\pi}{4}$.

(Also check for other values of m).

32. (d) We have $\sin A = \frac{4}{5}$ and $\cos B = -\frac{12}{13}$

Now, $\cos(A+B) = \cos A\cos B - \sin A \sin B$

$= \sqrt{1-\frac{16}{25}}\left(-\frac{12}{13}\right) - \frac{4}{5}\sqrt{1-\frac{144}{169}}$

$= -\frac{3}{5}\times\frac{12}{13} - \frac{4}{5}\left(-\frac{5}{13}\right) = -\frac{16}{65}$

33. (b) Given that $A + B = \frac{\pi}{4} \Rightarrow \tan(A+B) = \tan\frac{\pi}{4}$

$\Rightarrow \frac{\tan A + \tan B}{1-\tan A\tan B} = 1$

$\Rightarrow \tan A + \tan B + \tan A \tan B = 1$

$\Rightarrow (1+\tan A)(1+\tan B) = 2$.

34. (d) $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ}$

$$= \frac{2\left(\frac{1}{2}\cos 10^o - \frac{\sqrt{3}}{2}\sin 10^o\right)}{\frac{2}{2}\left(= \frac{\cos 10^o - \sqrt{3}\sin 10^o}{\sin 10^o \cos 10^o}\sin 10^o \cos 10^o\right)}$$

$= \frac{4\sin(30^o - 10^o)}{\sin 20^o} = \frac{4\sin 20^o}{\sin 20^o} = 4$.

35. (b) We have $\cos(\alpha+\beta) = \frac{4}{5}$

and $\sin(\alpha - \beta) = \frac{5}{13}$

$\Rightarrow \sin(\alpha+\beta) = \frac{3}{5}$ and $\cos(\alpha-\beta) = \frac{12}{13}$

$\Rightarrow 2\alpha = \sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13}$

$= \sin^{-1}\left[\frac{3}{5}\sqrt{1-\frac{25}{169}} + \frac{5}{13}\sqrt{1-\frac{9}{25}}\right]$

$\Rightarrow 2\alpha = \sin^{-1}\left(\frac{56}{65}\right) \Rightarrow \sin 2\alpha = \frac{56}{65}$

Now, $\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{56/65}{33/65} = \frac{56}{33}$.

36. (a) $\cos 15^o - \sin 15^o = \sqrt{2}.\cos(45^o + 15^o) = \sqrt{2}.\cos 60^o$

$= \sqrt{2}.\frac{1}{2} = \frac{1}{\sqrt{2}}$.

37. (a) We have $5x = 3x + 2x \Rightarrow \tan 5x = \tan(3x+2x)$

$\Rightarrow \tan 5x = \frac{\tan 3x + \tan 2x}{1 - \tan 3x \tan 2x}$

$\Rightarrow \tan 5x - \tan 5x \tan 3x \tan 2x = \tan 3x + \tan 2x$

$\Rightarrow \tan 5x \tan 3x \tan 2x = \tan 5x - \tan 3x - \tan 2x$.

38. (a) Divided by $\cos 17^o$ in numerator and denominator,

we get, $\dfrac{\cos 17^o + \sin 17^o}{\cos 17^o - \sin 17^o}$

$$= \frac{1+\tan 17^o}{1-\tan 17^o} = \frac{\tan 45^o + \tan 17^o}{1-\tan 45^o \tan 17^o} = \tan 62^o .$$

39. (b) $2\cos\dfrac{\pi}{13}.\cos\dfrac{9\pi}{13} + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$$= 2\cos\frac{\pi}{13}.\cos\frac{9\pi}{13} + 2\cos\frac{4\pi}{13}\cos\frac{\pi}{13}$$

$$= 2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13} + \cos\frac{4\pi}{13}\right]$$

$$= 2\cos\frac{\pi}{13}\left[2\cos\frac{\pi}{2}.\cos\frac{5\pi}{26}\right] = 0 , \qquad \left[\because \cos\frac{\pi}{2} = 0\right].$$

40. (d) $\cos\dfrac{\pi}{5}\cos\dfrac{2\pi}{5}\cos\dfrac{4\pi}{5}\cos\dfrac{8\pi}{5}$

$$= \frac{\sin\frac{2^4\pi}{5}}{2^4 \sin\frac{\pi}{5}} = \frac{\sin\frac{16\pi}{5}}{16\sin\frac{\pi}{5}} = \frac{\sin\left(3\pi + \frac{\pi}{5}\right)}{16\sin\frac{\pi}{5}}$$

$$= \frac{-\sin\frac{\pi}{5}}{16\sin\frac{\pi}{5}} = -\frac{1}{16}.$$

41. (c) $\dfrac{\sin 3\theta + \sin 5\theta + \sin 7\theta + \sin 9\theta}{\cos 3\theta + \cos 5\theta + \cos 7\theta + \cos 9\theta}$

$$= \frac{(\sin 3\theta + \sin 9\theta) + (\sin 5\theta + \sin 7\theta)}{(\cos 3\theta + \cos 9\theta) + (\cos 5\theta + \cos 7\theta)}$$

$$= \frac{2\sin 6\theta \cos 3\theta + 2\sin 6\theta \cos\theta}{2\cos 6\theta \cos 3\theta + 2\cos 6\theta \cos\theta}$$

$$= \frac{2\sin 6\theta(\cos 3\theta + \cos\theta)}{2\cos 6\theta(\cos 3\theta + \cos\theta)} = \tan 6\theta .$$

42. (b) $\cos A + \cos(240^o + A) + \cos(240^o - A)$

$$= \cos A + 2\cos 240^o \cos A$$

$$= \cos A\{1 + 2\cos(180^o + 60^o)\} = \cos A\left\{1 + 2\left(-\frac{1}{2}\right)\right\}$$

$$= 0 .$$

43. (b) $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{a+b}{a-b}$

$$\Rightarrow \frac{\sin(x+y) + \sin(x-y)}{\sin(x+y) - \sin(x-y)} = \frac{(a+b)+(a-b)}{(a+b)-(a-b)}$$

$$\Rightarrow \frac{2\sin x\cos y}{2\cos x\sin y} = \frac{2a}{2b} \Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b}.$$

44. (a) $\cos^2\dfrac{\pi}{12} + \cos^2\dfrac{\pi}{4} + \cos^2\dfrac{5\pi}{12}$

$$= 1 - \sin^2\left(\frac{\pi}{12}\right) + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\left(\frac{5\pi}{12}\right)$$

$$= 1 + \frac{1}{2} + \left(\cos^2\frac{5\pi}{12} - \sin^2\frac{\pi}{12}\right)$$

$$= \frac{3}{2} + \cos\left(\frac{5\pi}{12} + \frac{\pi}{12}\right)\cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right) = \frac{3}{2} + \cos\frac{\pi}{2}\cos\frac{\pi}{3}$$

$$= \frac{3}{2} + 0.\frac{1}{2} = \frac{3}{2}.$$

45. (a) $\cos^2\alpha + \cos^2(\alpha + 120^o) + \cos^2(\alpha - 120^o)$

$= \cos^2\alpha + \{\cos(\alpha + 120^o) + \cos(\alpha - 120^o)\}^2$

$- 2\cos(\alpha + 120^o)\cos(\alpha - 120^o)$

$= \cos^2\alpha + \{2\cos\,\alpha\cos 120^o\}^2 - 2\{\cos^2\alpha - \sin^2 120^o\}$

$= \cos^2\alpha + \cos^2\alpha - 2\cos^2\alpha + 2\sin^2 120^o$

$= 2\sin^2 120^o = 2\times\frac{3}{4} = \frac{3}{2}.$